Learning Renormalization Group Flows

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The Big Question

Real Space Renormalization is a powerful, but practically difficult technique in statistical physics.

Can we automate it?

1. What is Real Space Renormalization?
2. Real Space Renormalization of the Ising Model
3. Results

At a high level...

Renormalization is a theory of theories.

Guiding Question

How does our model for a system change as we consider it from different **length scales**?

A Toy Example

Say we want to simulate a drop of water. What does this system look like?

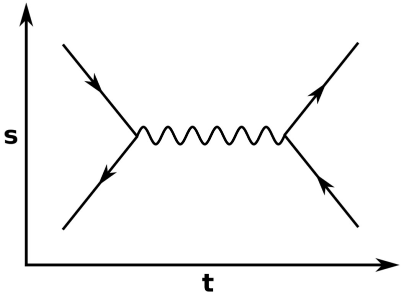
- $\sim 10^{19}$ water molecules
- 3 atoms in each molecule
- A bunch of fundamental particles in each atom

What are the models we have to understand such a system?

A Toy Example: Smallest Scale Model

At the particle scale, field theories:

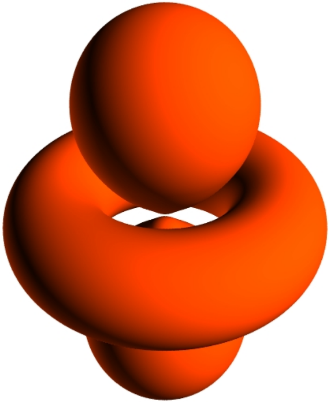
- Incredibly complex model with many variables and physical quantities to keep track of
- Practically intractable integrals to compute
- Completely infeasible to simulate even a drop of water



A Toy Example: Smaller Scale Model

At the subatomic scale, quantum dynamics:

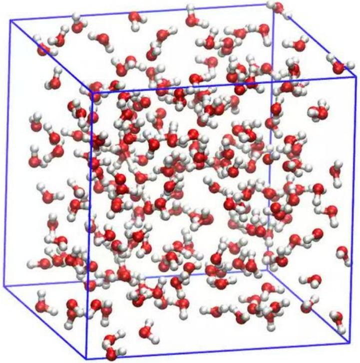
- Complex model with fewer, but still many variables and physical quantities
- Difficult integrals to compute
- Simulation is possible, but still intractable in general



A Toy Example: Small Scale Model

At the atomic scale, classical dynamics:

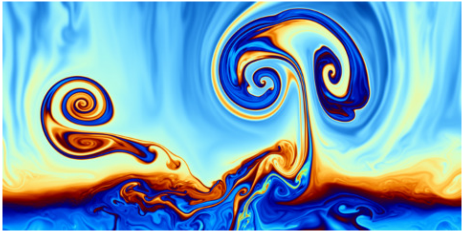
- Intuitive model with only a couple of degrees of freedom and parameters per molecule
- Integrals are easy
- Simulation is expensive, but doable for up to decently large systems



A Toy Example: Large Scale Model

At the human scale, fluid dynamics:

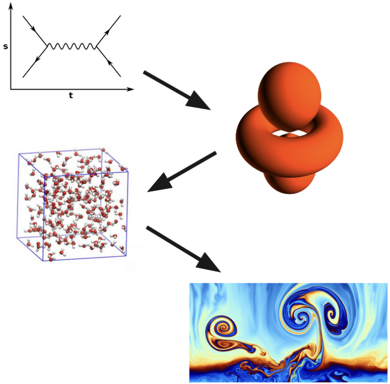
- Simple, elegant theory modeled by a single PDE
- A few quantities (viscosity, temperature, pressure, ...) fully specify the system
- Simulation practical and accurate up to very large volumes



Renormalization is a Theory of Scaling

Real Space Renormalization is a theory of how these models, all at different scales, can be linked in a highly systematic way.

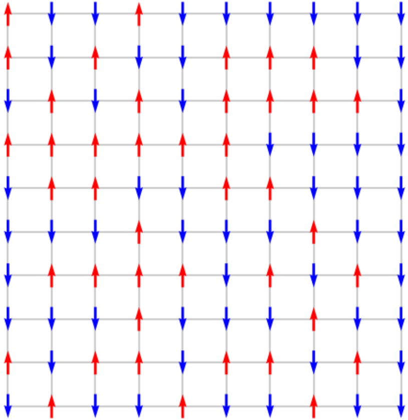
As it turns out, such an understanding of scale is dual to an understanding of the system's fundamental thermodynamics.



The Ising Model

Here, the specific system we will study is the Ising model for ferromagnetism:

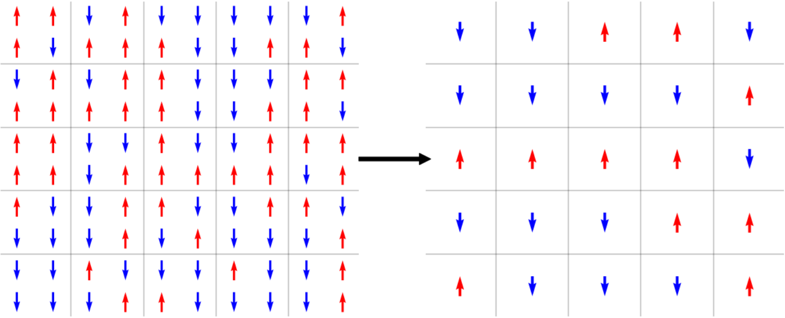
- Atoms are arranged on some lattice
- Atomic spins are either up or down, and are correlated to neighboring spins
- The correlation has strength J.



The "Zoomed Out" Ising Model

To study the Ising model at a larger scale, we "zoom out" by approximating a block of microscopic spins with a single macroscopic spin.

To compensate, the correlation strength J will change.



The Renormalization Group Flow

We want to know exactly how J responds to "zooming out".

That is, we would like to find a function $f(J)$, called the renormalization group (RG) flow, that describes the change in J upon "zooming out".

Analytically solving for the Ising model's $f(J)$ is extremely hard—physicists have been trying since the 1970's.

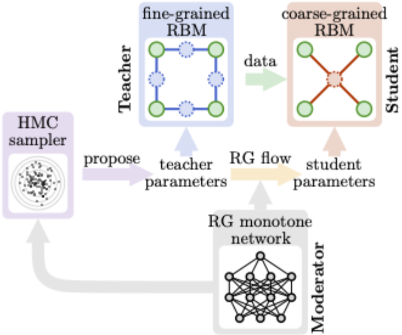
Recently, Hou et al. proposed an algorithm that models $f(J)$ using machine learning.

Deep Learning Procedure

The algorithm works by modeling $f(J)$ with a neural network $f_\theta(J)$.

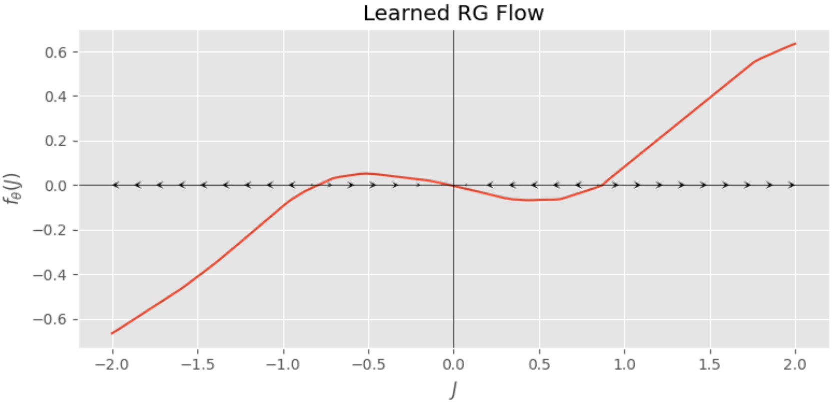
We want to adjust $f_\theta(J)$ to best resemble to appropriate flow $f(J)$.
To do so, we use a supervised learning procedure:

1. **Sample** values of J.
2. **Forward Pass** to evaluate $f_\theta(J)$.
3. **Estimate the Error** between $f_\theta(J)$ and $f(J)$.
4. **Backward Pass** to adjust $f_\theta(J)$ according to that error.



Learned RG Flow

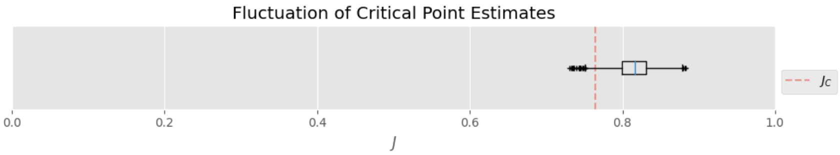
Following this procedure, we train a neural network $f_\theta : \mathbb{R} \to \mathbb{R}$.
Upon convergence, f_θ exhibits the desired fixed and critical points:



Critical Point Estimates

Given the learned f_θ function, we use Newton's method to numerically solve for fixed and critical points where $||f_\theta|| \to 0$

The values vary, but we can plot their distribution by training many models:

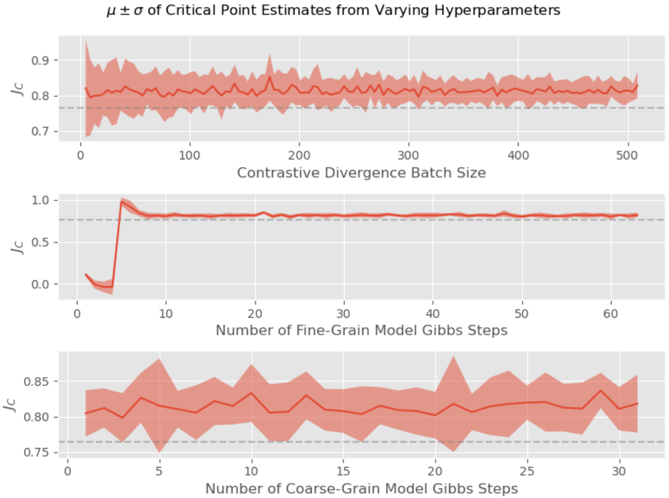


Evidently, our estimates are biased from the analytically derived results. Why?

Reducing Bias from Contrastive Approximation

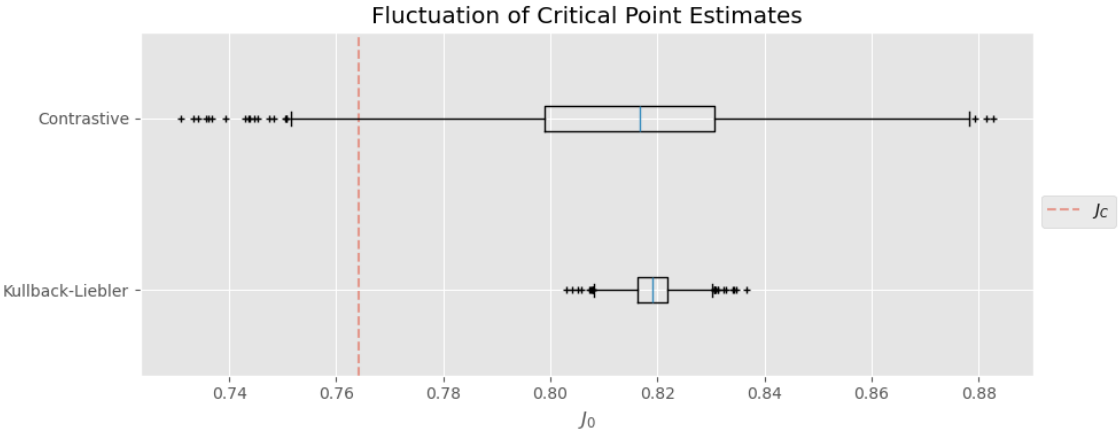
One possible source of bias is the use of approximations when evaluating the error.

We can refine approximations with more computation, but this only decreases bias up to a certain extent.



Reducing Bias from Contrastive Approximation

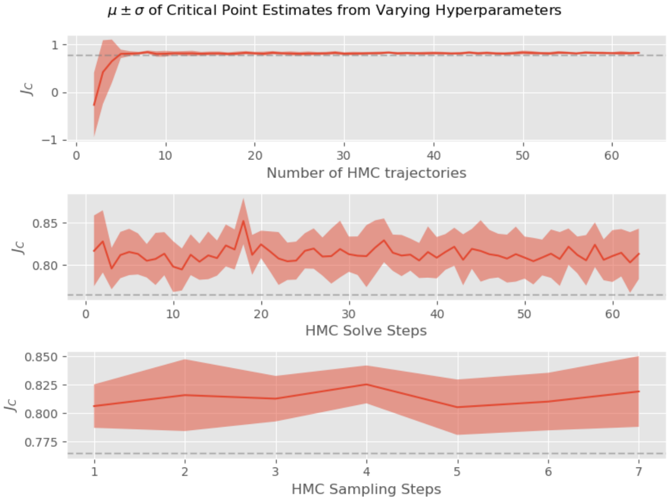
In fact, even computing the exact error doesn't reduce the bias, only serving to reduce variance:



Reducing Bias from Sampling Inefficiency

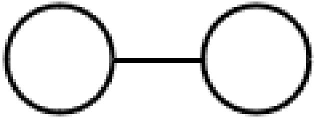
The other possible source of random bias is some inefficiency in the sampling procedure.

Experimenting with a range of sampler settings shows only limited effects on the bias.

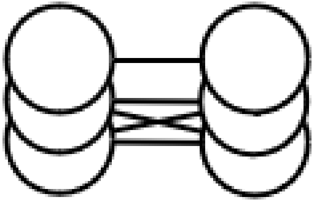


Reducing Bias by Increasing Representation Complexity

Hou et al. conjecture that the bias results from the simplicity of the Ising model, i.e. the blocking approximations are too coarse.



To mitigate this, they propose stacking additional spins and correlation constants, increasing the complexity and expressiveness of the Ising model.



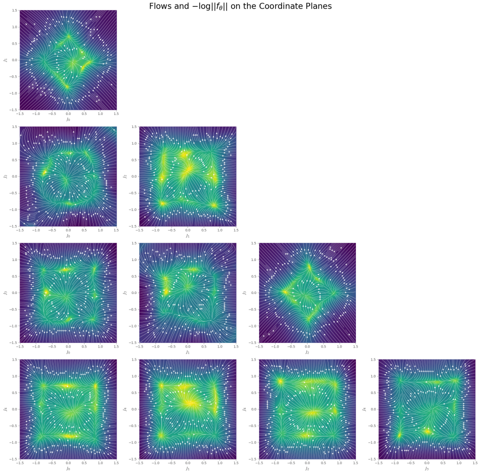
This means we need to model an n-dimensional flow:

$$\vec{f}_\theta(\vec{J}) : \mathbb{R}^n \to \mathbb{R}^n$$

Higher Dimensional Flows

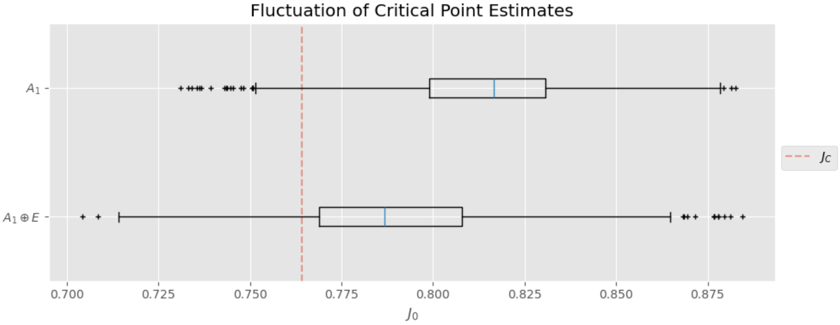
Using the algorithm, we model a flow over 5 correlation constants.

$$\vec{f}_\theta(\vec{J}) : \mathbb{R}^5 \to \mathbb{R}^5$$



Reduced Bias from Increased Representation Complexity

The estimates of the critical points of this 5-dimensional flow exhibit reduced bias:



Hou et al. show that this trend continues as the flow dimension increases.

Conclusions

This algorithm presents a new, exciting way to analyze statistical systems using an automated real space renormalization approach.

Estimates are biased, but we show the bias can be mitigated by tuning the training procedure and increasing representational complexity.

In the future, there are many exciting avenues to be explored:

- What other sources of bias can we identify?
- How can we improve the algorithm for more efficient modeling of high-dimensional flows?
- What systems beyond square Ising models can we characterize using this method?

Acknowledgements

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